Advanced Algorithms. Home Exam 2025/26

The Problem

We are given an undirected graph $G = (V, E)$. Its node set V is partitioned into so called clusters C_i $(i = 1, \ldots, n)$ which are pairwise disjoint. That is, $V = C_1 \cup \ldots \cup C_n$, and $C_i \cap C_j = \emptyset$ holds for all pairs of indices $i \neq j$. An edge uv from E is called internal if its nodes belong to the same cluster ($u, v \in C_i$ for some i) and external if its nodes belong to different clusters ($u \in C_i$ and $v \in C_j$ for some $i \neq j$). Every edge has some positive cost. The problem is to find some cheapest path P that traverses at least one internal edge from every cluster.

Some clarifications: P may visit nodes and edges repeatedly and traverse the same edges in both directions. P is not necessarily a cycle; its two end nodes can be distinct. The cost of P is the sum of costs of the traversed edges (where repeatedly traversed edges count multiple times).

Task: *Devise some algorithm(s) for this problem that yield exact or approximate solutions, and analyze their running times and (if appropriate) their approximation ratios.*

The problem appears in practice, when a complicated surface shall be treated (by milling, painting, etc.) by some device, e.g., a robot hand. The clusters represent simple parts of the surface, the internal edges represent alternative precomputed simple paths that cover these parts, and the external edges represent possible movements between the ends of those paths.

Some suggestions for possible directions of your work

- Can you show NP-completeness of the problem?
- Can you apply dynamic programming on subsets to get exact solutions? If so, how?
- Some self-study of approximation algorithms for related problems may be inspiring, or even necessary. Some keywords for searching: Christofides' heuristic, Chinese postman, rural postman, embroidery problem.
- In the mentioned application, internal edges (that cover entire areas) are much more expensive than external edges (that just provide short links). How does this affect the approximation ratios? Can you quantify this somehow?
- The internal edges within the same cluster may always have similar costs. How beneficial is this for the approximation ratio?
- Does it play a role whether the costs satisfy the triangle inequality?
- Does something change if there are additional turn costs between the edges?
- ... and any other related thoughts you might have.

Submission

- Upload your final report to the Fire system, as a PDF document (no other formats please!). It is also recommended to additionally mail the identical document to `ptr@chalmers.se`.
- Given the complexity of the task and the need to rewrite drafts, handwriting is discouraged.
- Write your name and study programme on the title page. The final submission deadline is announced on the course web page. Do not wait until the last minute, but submit when you are done. See also the Instructions below.
- Quality is more important than quantity. However, as a rule of thumb, your report should have at least 4 pages of text (with reasonable font size, spacing, and margins), plus the title page and possible references.

Criteria for a Good Report

- Correctness: There are no major factual mistakes and invalid calculations. The final "product" does not have to be perfect or contain brilliant results, but what you write must be sound.
- Depth: You provide some solid, substantial results that are fully worked out, not only trivial observations or vague heuristic guesses.
- Clarity: All notations are well defined. Algorithms, as well as proofs of their properties, are well described. One can follow your arguments step by step without extra efforts. See also the general grading criteria on the course web page.
- Negative statements (that something is probably not possible to do) are motivated by good reasons.

Instructions

- You have some freedom to choose your specific working directions, as long as you stick to the given problem. Do not misunderstand the suggestions as exercises that must all be solved.
- Only a certain selection of concepts and tools from the course are suitable for this problem while most others are probably not, and it is part of the task to figure out what fits here – which would also be the typical situation in practice.
- Do not hesitate to discuss, and to send questions or drafts. But take availability times into account; see the course web page. Do not expect feedback on the drafts that goes much into technical details; this would not be manageable. The purpose is only to provide some initial guidance and to check whether you are on a meaningful track.
- You can send arbitrarily many drafts, at any time. In fact, **drafts are strongly encouraged**. The worst strategy would be to start thinking only a few days before the deadline – this would most likely result in a very poor report, because the details may be more tricky than anticipated at first glance.
- Do not upload drafts in Fire, but only your final submission.

- Utmost academic honesty is expected. The words about cheating (see the course web page) apply also here. In particular, you must cite all literature you have used, acknowledge all sources of help (which also includes AI tools), and always describe the contents in your own way. Also mark very clearly in the text what is taken from other sources and does not come from your own thoughts.